Stats 2 – May / June 2006

(1) a) $X \sim Po(3)$

For a 5 year period:

$Y \sim Po(15)$

$P(Y > 18) = 1 - P(Y \leq 18)$

$= 1 - 0.8195$ From Tables!

$= 0.1805$

b) $Y \sim Po(7)$

i) $(X + Y) \sim Po(10)$

$P(X+Y) < 15 = P(X+Y) \leq 14$

$= 0.9165$ From Tables

ii) That Maths grades and English grades are independent

(2) a) From calculator:

$n = 5$ $\quad \bar{x} = 50.8$

$\Sigma x = 254$ $\quad s = 4.5497...$

$\Sigma x^2 = 12986$ $\quad s^2 = 20.7$

$v = 5 - 1 = 4$

don't know pop variance so must use t

t critical value (95%) $= 2.776$

Confidence interval : $50.8 \pm 2.776 \times \frac{4.5497..}{\sqrt{5}}$

$= (45.151.., 56.448...)$

b) 5% or 0.05 as 95% CI

(3) a) $E(X) = 1 \times (7/16) + 2 \times (5/16) + 3 \times (3/16) + 4 \times (1/16)$

$= 15/8 = \text{mean}$

$E(X^2) = 1^2 \times (7/16) + 2^2 \times (5/16) + 3^2 \times (3/16) + 4^2 \times (1/16)$

$= 35/8$

Variance $= E(X^2) - [E(X)]^2 = 35/8 - (15/8)^2$

$= 55/64$

b) i) $32 \times (3/16) + 32 \times (1/16) = 8$ games

ii) when score at least 3 $= 0.9 \times 8 = 36/5$

when score 2 goals $= 0.5 \times [32 \times (5/16)] = 5$

when score 1 goal $= 0.2 \times [32 \times (7/16)] = 14/5$

TOTAL $= \underline{\underline{15}}$

4) a) i)

	A	B	
22-34	21	32	53
35-39	72	36	108
40-59	27	12	39
	120	80	$\boxed{200}$

ii) H_0: No association between area and age (Independent)

H_1: Association between area and age (Non-Independent)

<u>Expected</u>

	A	B
22-34	31.8	21.2
35-39	64.8	43.2
40-59	23.4	15.6

<u>X^2 values</u> $\sum \frac{(O_i - E_i)^2}{E_i}$

	A	B
22-34	3.6679	5.5019
35-39	0.8	1.2
40-59	0.5538	0.8308

$\sum X^2 = 12.554$ ~~(CRITICAL VALUE)~~ (TEST STATISTIC)

~~TEST STATISTIC : X~~

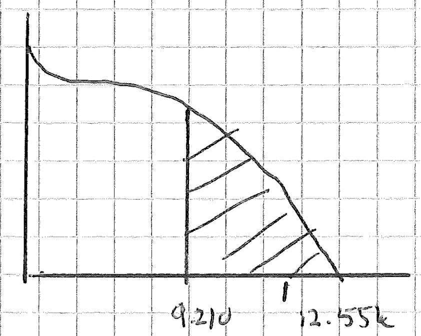
$v = (3-1)(2-1) = 2$

CRITICAL VALUE : $X^2_{1\%}(2) = 9.210$

$12.554 > 9.210$

$\therefore$ Reject H_0

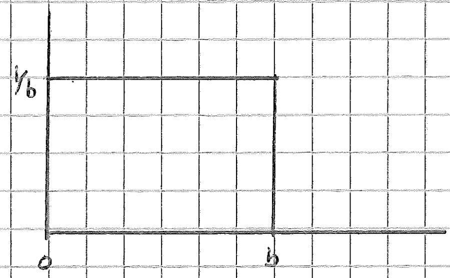
There is evidence of an association between area and age profile



b) In school A, fewer staff employed than expected in 22-34 group

In school B, more staff employed than expected in 22-34 group

5 a) i) $E(X) = b/2$



ii) $E(X^2) = \int x^2 f(x)$

$= \int_0^b x^2 (1/b)$

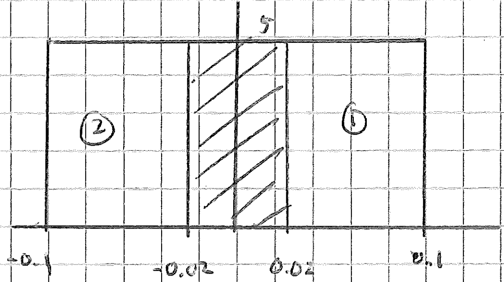
$= 1/b \int_0^b x^2$

$= 1/b \left[x^3/3 \right]_0^b = 1/b \left[b^3/3 - 0 \right] = b^2/3$

$Var(X) = E(X^2) - E(X)^2$

$= b^2/3 - (b/2)^2 = b^2/3 - b^2/4 = 1/12\, b^2$

b)



$P(|T| > 0.02) = (1) + (2)$ or $(1) \times 2$

$= \left[5 \times (0.1 - 0.02) \right] \times 2$

$= 4/5$ or 0.8

6 a) $H_0: \mu = 100$

$H_1: \mu < 100$ (1 tailed)

Use t-distributions as we don't know pop variance

Data from calc:

$n = 5$ $\bar{x} = 94.2$

$\Sigma x = 471$ $s = 6.0580.$

$\Sigma x^2 = 44,515$ $s^2 = 36.7$

$v = 5 - 1 = 4$

TEST STATISTIC: $\dfrac{94.2 - 100}{6.0580/\sqrt{5}} = -2.140...$

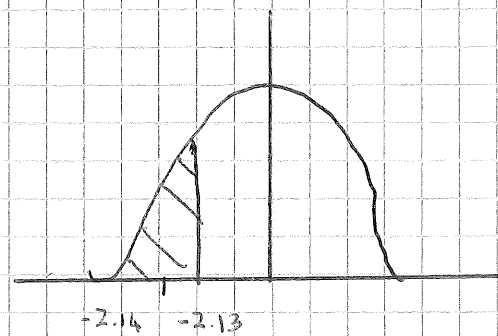
~~CRITICAL VALUE~~

CRITICAL VALUE: $t_{5\%}(4)$ 1 tailed : -2.132

$-2.14 < -2.13$

$\therefore$ Reject H_0

There is evidence at 5% level to Reject H_0 and support the view that the average life of batteries is less than 100 hours



b) As the sample is now $n = 80$, we can use the Z distribution because of Central Limit Theorem

$H_0: \mu = 100$

$H_1: \mu \neq 100$ (2 tailed test)

$\bar{x} = \frac{8080}{80} = 101$

$s^2 = \frac{6399}{79} = 81 \qquad s = 9$

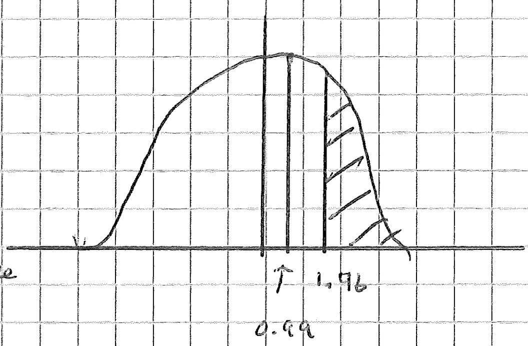
Test Statistic: $Z = \frac{101 - 100}{9/\sqrt{80}} = 0.9938\ldots$

CRITICAL VALUE: 5% (2 TAILED) $= \pm 1.96$

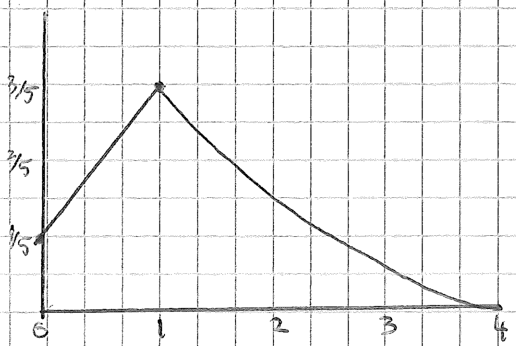
$0.99 < 1.96$

$\therefore$ Accept H_0

There is enough evidence at 5% level to accept the manufacturer's claim that the mean battery life is 100 hours



7 a)



b) i) $F(x) = \int_0^x f(x)$

$= \int_0^x \frac{1}{5}(2x+1)\,dx$

$= \frac{1}{5}\int_0^x 2x + 1 \,dx$

$= \frac{1}{5}\left[x^2 + x\right]_0^x$

$= \frac{1}{5}x^2 + \frac{1}{5}x - 0$

$= \frac{1}{5}x(x+1)$

ii) $P(X \leq 1) = F(1) = \frac{1}{5}(1)(1+2) = \frac{2}{5}$

iii) $P(X \geq x) = 1 - P(X \leq x)$

so $\frac{17}{20} = 1 - P(X \leq x)$

so $P(X \leq x) = \frac{3}{20}$

so $F(x) = \frac{3}{20}$

$\frac{1}{5}x(x+1) = \frac{3}{20}$

$x(x+1) = {}^{15}/_{20}$

$x^2 + x = {}^{15}/_{20}$

$20x^2 + 20x = 15$

$20x^2 + 20x - 15 = 0$

$4x^2 + 4x - 3 = 0$

$(2x - 1)(2x + 3) = 0$

$\downarrow$

$x = 1/2$ or $x = -3/2$

so $x = 1/2$ as $0 \le x \le 4$

iv) As $F(1) = 0.4$, the LQ is $0 \le x \le 1$

$F(x) = 0.25$

$\frac{1}{5}x^2 + \frac{1}{5}x = 0.25$

$x^2 + x - 1.25 = 0$

Doesn't factorise so use formula:

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-1.25)}}{2}$$

$$= \frac{-1 \pm \sqrt{6}}{2} = \frac{1}{2}(-1 + \sqrt{6})$$ as

x must be positive.